STELAR: Spatio-temporal Tensor Factorization with Latent Epidemiological Regularization

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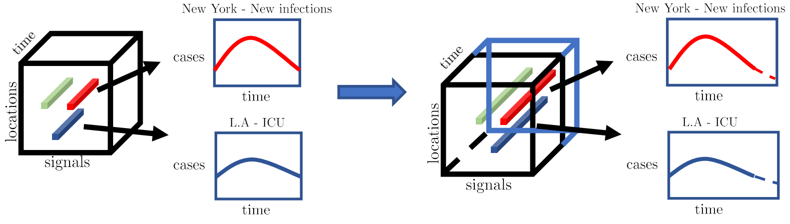
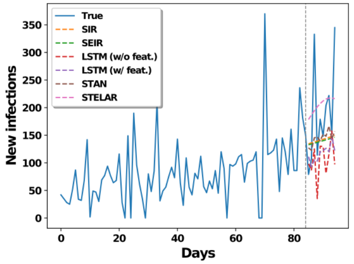
Epidemic Prediction

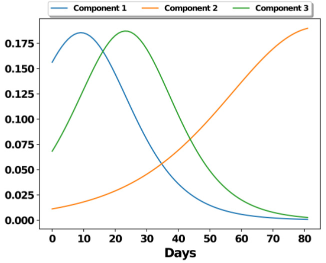
- **Pandemic diseases**
  - Serious threat to public heath, economy and daily life.
  - Accurate measurement, modeling and tracking are needed.
  - Effective mitigation measures.
- **Task**
  - Case counts for different locations and signals over time.
  - Prediction of epidemic trends for all locations simultaneously.



In a Nutshell

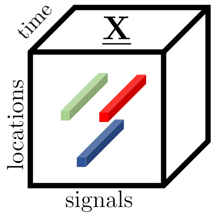
- **In this work we:**
  - Propose STELAR, a data efficient tensor factorization method to predict the evolution of epidemic trends.
  - Perform experiments on real county- and state-level COVID-19 data.
  - Demonstrate superior prediction performance compared to baselines.
  - Identify interesting latent patterns of the epidemic.
  - Code: https://github.com/nkargas/STELAR





STELAR

- **M locations**
  - Counties/states in the US.
- **N signals**
  - Daily new infections, ICU patients, …
- **L timesteps**



Low-rank CPD model with SIR constraints on the latent time factor

$$\underline{\mathbf{X}} = [\![\mathbf{A}, \mathbf{B}, \mathbf{C}]\!] \qquad \begin{aligned} c_{t,k} &= \beta_k S_k(t-1) I_k(t-1) \\ S_k(t) &= S_k(t-1) - \beta_k S_k(t-1) I_k(t-1) \\ I_k(t) &= I_k(t-1) + \beta_k S_k(t-1) I_k(t-1) - \gamma_k I_k(t-1) \\ s_k &= S_k(0), i_k = I_k(0) \end{aligned}$$

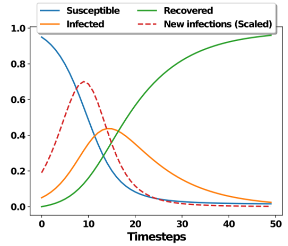
SIR Model

- Susceptible S(t), infected I(t) and recovered R(t) subpopulations.

$$S(t) - S(t-1) = -\beta S(t-1) I(t-1)/N$$
$$I(t) - I(t-1) = \beta S(t-1) I(t-1)/N - \gamma I(t-1)$$
$$R(t) - R(t-1) = \gamma I(t-1)$$

- New infections

$$C(t) := \beta S(t-1) I(t-1)/N$$

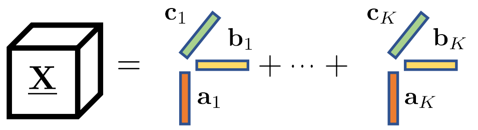


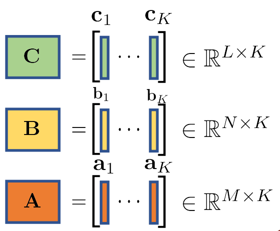
Canonical Polyadic Decomposition (CPD)

- A 3-way tensor admits a decomposition of rank K if it can be decomposed as a sum of K rank-1 tensors.

$$\underline{\mathbf{X}} = [\![\mathbf{A}, \mathbf{B}, \mathbf{C}]\!]$$

$$\underline{\mathbf{X}} = \sum_{k=1}^{K} \mathbf{a}_k \circ \mathbf{b}_k \circ \mathbf{c}_k$$





Problem Formulation

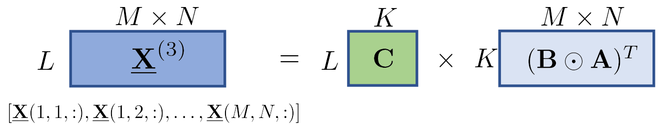
$$\min_{\substack{\mathbf{A},\mathbf{B},\mathbf{C},\\ \boldsymbol{\beta},\boldsymbol{\gamma},\mathbf{s},\mathbf{i}}} \|\underline{\mathbf{X}} - [\![\mathbf{A}, \mathbf{B}, \mathbf{C}]\!]\|_F^2 + \mu\left(\|\mathbf{A}\|_F^2 + \|\mathbf{B}\|_F^2 + \|\mathbf{C}\|_F^2\right) + \nu \sum_{k=1}^{K}\sum_{t=1}^{L} \left(c_{t,k} - \beta_k S_k(t-1) I_k(t-1)\right)^2$$

$$\text{s. t.} \quad \mathbf{A} \geq \mathbf{0}, \mathbf{B} \geq \mathbf{0}, \mathbf{C} \geq \mathbf{0}, \quad \boldsymbol{\beta} \geq \mathbf{0}, \boldsymbol{\gamma}, \geq \mathbf{0}, \mathbf{s} \geq \mathbf{0}, \mathbf{i} \geq \mathbf{0}$$

$$S_k(t) = S_k(t-1) - \beta_k S_k(t-1) I_k(t-1),$$
$$I_k(t) = I_k(t-1) + \beta_k S_k(t-1) I_k(t-1) - \gamma_k I_k(t-1),$$
$$s_k = S_k(0), i_k = I_k(0).$$

- **Prediction**
  - Our model expresses the evolution of a signal as a **weighted sum of K separate SIR models**.
  - Captures **correlations** between different **locations and signals** through their latent representations.

$$\underline{\mathbf{X}}(m,n,t) = \sum_{k=1}^{K} a_{m,k} b_{n,k} c_{t,k} = \sum_{k=1}^{K} a_{m,k} b_{n,k} \beta_k S_k(t-1) I_k(t-1).$$



Results

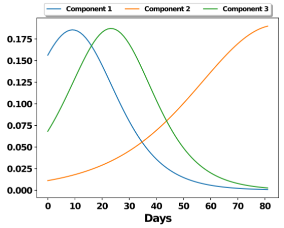
New infections

Model	$L_o = 10$ RMSE	$L_o = 10$ MAE	$L_o = 15$ RMSE	$L_o = 15$ MAE
Mean	304.1	122.0	269.5	108.5
SIR	156.2	62.2	159.1	63.6
SEIR	177.1	72.9	163.2	69.7
LSTM (w/o feat.)	203.6	77.1	191.0	81.7
LSTM (w/ feat.)	162.3	68.2	187.6	78.3
STAN	164.2	61.1	152.6	61.8
STELAR ($\nu = 0$)	149.2	61.5	152.8	66.9
STELAR	**127.5**	**55.6**	**136.1**	**61.7**

Hospitalized patients

Model	$L_o = 10$ RMSE	$L_o = 10$ MAE	$L_o = 15$ RMSE	$L_o = 15$ MAE
Mean	125.0	77.0	123.3	77.1
SIR	46.5	27.2	48.7	27.7
SEIR	39.1	23.9	41.1	25.7
LSTM (w/o feat.)	45.6	23.6	54.8	31.2
LSTM (w/ feat.)	42.5	23.3	47.5	26.8
STAN	30.6	17.3	42.8	24.2
STELAR ($\nu = 0$)	28.6	16.6	46.8	21.1
STELAR	**24.0**	**15.1**	**36.0**	**18.0**

Interpretability



Component 1	Component 2	Component 3
New York (NY)	L.A (CA)	Nassau (NY)
Westchester (NY)	Cook (IL)	L.A (CA)
Nassau (NY)	Milwaukee (WI)	Essex (NJ)
Bergen (NJ)	Fairfax (VA)	Wayne (MI)
Miami-Dade (FL)	Hennepin (MN)	Oakland (MI)
Hudson (NJ)	Montg. (MD)	Middlesex (NJ)
Union (NJ)	P. George's (MD)	New York (NY)
Phila. (PA)	Dallas (TX)	Phila. (PA)
Passaic (NJ)	Orange (CA)	Cook (IL)
Essex (NJ)	Harris (TX)	Bergen (NJ)